Apparatus for the Determination of Gravity at Sea.

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The apparatus to be described below was designed by the writer for the purpose of estimating the value of gravity at sea. In an experimental stage the instrument has been taken to Australia and back in the refrigerator of a ship, together with other apparatus designed for the purpose, and its behaviour as regards pumping and temperature variation carefully noted. The preliminary tests justify the continuance of an investigation of the value of gravity by this method.

The following account of the instrument and its performance during its preliminary test has been compiled with a view to indicating the improvements which may subsequently be made in the instrument and its method of use.

The general principle of the apparatus was that adopted by Hecker,* in some instruments which he had constructed but had not tested, in which a column of mercury was balanced against the pressure of air contained in the enclosed reservoir of a barometer, which was maintained at a nearly constant temperature. From a knowledge of the capacity correction of the reservoir and from visual or photographic observations of the upper level of the mercury, the length of the column could be calculated, the fluctuations in the temperature of the reservoir and stem being duly allowed for.

In the apparatus described below, the volume of air in the reservoir was maintained constant, but the change in the length of the column of mercury was not measured directly, but calculated from the volume of mercury which it was necessary to introduce in order to obtain a balance. Temperature corrections had still to be made but may be almost completely obviated in future work, as will be shown later.

The development of the apparatus is of interest. It arose from a discussion with Mr. Horace Darwin, of the Cambridge Scientific Instrument Company, who had designed an apparatus for the measurement of g upon the same principle as that underlying Hecker's apparatus, but in which the volume of air in the barometer reservoir was maintained constant, and the adjustment made by straining the glass of which the apparatus was constructed (the vertical column was curled into a spiral for this purpose). As far as the writer is aware, the form of apparatus devised by Darwin has not been constructed; it might very well prove of value in estimations of g on land areas, but at sea the spiral is too liable to pump up and down with the ship's motion, in addition to the pumping of the mercury through it.

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* It was the loan of these instruments to me by Profs. Helmert and Hecker which first drew my attention to this method; Mascart, 'Comptes Rendus,' vol. 95, pp. 126 and 631 (1882), had, however, attempted measurements on land at an earlier date by balancing a column of mercury against the pressure of a mass of air.—G. D., July, 1916.

Messrs. Cossor have constructed for the writer two types of apparatus in which a constant volume of air at a known (and, if possible, fixed) temperature supports a column of mercury whose height depends upon the gravitational attraction, but they differ in the manner of their adjustment. In the first form, of which an example is shown in fig. 1, the level of the mercury in the base of the air reservoir was maintained at a fixed mark, by raising or lowering, by means of a magnet held outside the tube, an iron needle immersed in the mercury in the upper part of the apparatus. This lowered or raised the level of the mercury in the vertical tube, and so produced the change in pressure necessary to restore the lower surface of the mercury to its normal level. The value of gravity is measured by the length of the needle immersed. The diameters of the needle and the tube were so adjusted that the withdrawal of the needle 3 cm. gave the range in the variation of g encountered in passing from the latitude of London to the Equator. By reducing the diameter of the needle, the apparatus may be made more sensitive.

This form of the apparatus was abandoned because great difficulty was experienced in moving the needle up and down by the required amount, even when a lump of iron was attached to it. It moved in jerks and the adjustment could not be made with adequate delicacy.

Fig. 2 illustrates the second form of the apparatus, whose principal dimensions are given in Table I, together with other details to which reference will be made later. A constant volume of air was maintained in the bulb B by keeping the mercury always up to the pointer at C. The air in the bulb was under reduced pressure in order to keep the apparatus within reasonable dimensions. The barometer tube was bent so that H was vertically above C, the length of the column of mercury HC being approximately 20 cm. The level was kept at C by raising or lowering the level of the mercury in the index tube D, which was of fine bore, an operation which was effected by means of an exhaust pump. It was from the reading of the level of the mercury in this tube, when contact was made with the pointer at C, that the value of gravity was calculated. The other side tube E was of wider bore, and was used only for making the initial adjustment at the beginning of the voyage. It was introduced on account of the great difficulty of correctly adjusting the amount of mercury in the apparatus, and to enable it to be used for various ranges of temperature.

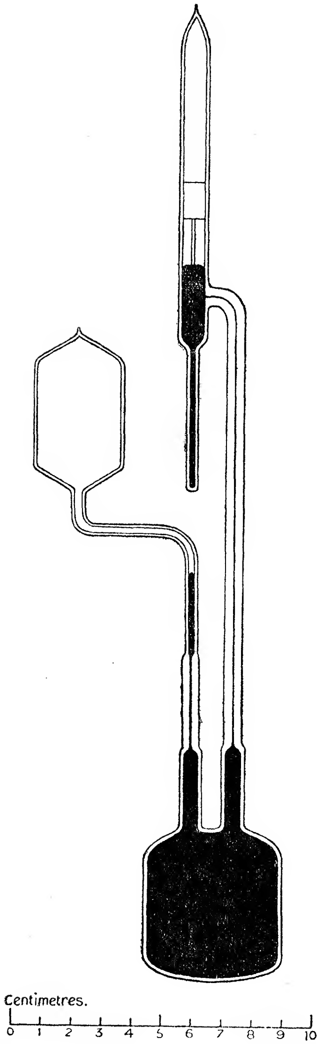

Fig. 1.—Preliminary form of the Apparatus.

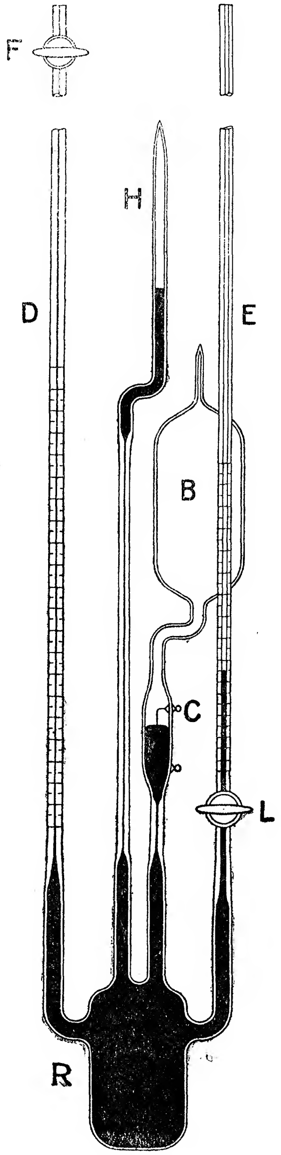

Fig. 2.—The instrument upon which the test was carried out.
(The graduations upon the tube D are in centimetres and half centimetres.)

Table I.—Dimensions of the Apparatus.

Approximate height of mercury column	$= h$	$= 20$ cm.
Diameter of top of barometer tube		$= 0·468$ cm.
Area	$= a$	$= 0·172$ sq. cm.
Diameter of contact tube		$= 0·99$ cm.
Area	$= b$	$= 0·769$ sq. cm.
Diameter of index tube		$= 0·051$ cm.
Area		$= 0·002$ sq. cm.
Diameter of rough adjustment tube		$= 0·15$ cm.
Area of capillary		$= 0·00064$ sq. cm.
Volume of air in reservoir		$= 100$ c.c.
Volume of mercury in instrument as originally constructed After reconstruction in Australia this was greatly exceeded, as it was found impracticable to reproduce a bulb of the original size.		$= 80$ c.c.
Length of capillary in barometer limb		$= 18$ cm.
Length of capillary in contact limb		$= 2$ cm.

From these dimensions we calculate that a change in level of 1 mm. in the index tube is equivalent to a change in the value of gravity of 0·058 cm./sec.^{2}

Between Lat. 0° and 54° a movement of the mercury in the index tube of about 60 mm. is required.

The reservoir of mercury R was introduced for the purpose of reducing the effect of temperature upon the readings, this device having been adopted in the apparatus suggested by Mr. Horace Darwin. By suitably designing the dimensions of the apparatus the increased pressure of air in bulb B due to a given rise of temperature is automatically counterbalanced by the rise of the level H occasioned by the expansion of the appropriate quantity of mercury in the reservoir. Though the compensation is only perfect at one temperature, the error at neighbouring temperatures is reduced to a minimum.

As it was necessary to maintain the whole apparatus at as uniform a temperature as possible, it was immersed in water which was contained in a stout zinc vessel furnished with the necessary supports, holes, and glass windows (fig. 3). The glass stopcocks were turned by clips engaging them, and operated through corks from outside, an arrangement which worked well. The tank was suspended by cords from the ceiling of the refrigerator of the vessel in which these preliminary experiments were conducted. The motion of the vessel kept the water stirred to some extent, and before making readings it was further agitated by pumping air through it by means of a bicycle pump, but in future work it would be well to stir the water continuously by means of a vane driven by a motor. From the writer's experience of apparatus in a refrigerator, it is more satisfactory to attempt to keep the whole at a uniform temperature than to attempt to measure and correct for the temperatures of the various parts of an exposed barometer separately, however carefully they may be lagged.

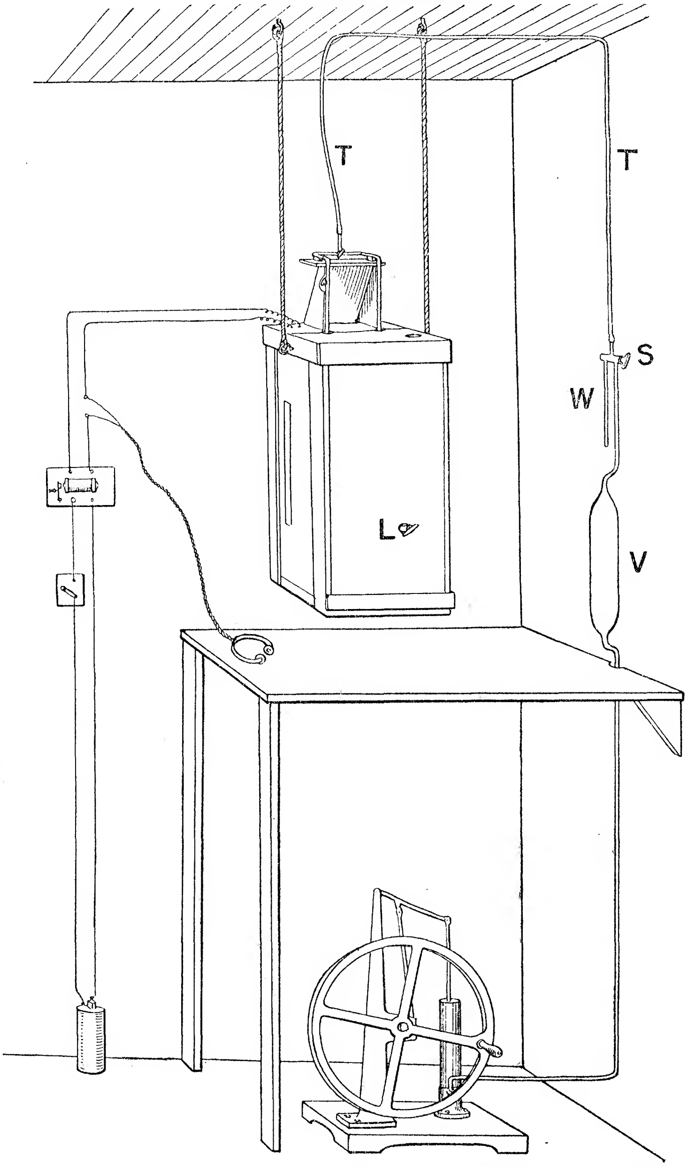

FIG. 3.—The arrangement of the apparatus in the refrigerator of R.M.S. 'Morea.'

The suspension not only kept the apparatus vertical as the ship rolled or pitched, but it diminished the disturbance of the surface of the mercury at C, occasioned by the throbbing of the engines and the vibration of the ship.

Capillary tubing was introduced to prevent the pumping of the mercury due to the vertical motion of the ship, the size of the bore being chosen after preliminary experiments in the Physics Laboratory of University College, Reading.

The moment of contact of the mercury with the pointer was detected electrically; for this purpose the pointer was connected to a terminal outside the glass, and another terminal sealed into the mercury reservoir, a trembling coil and telephone completing the circuit. The receiver was fitted over the observer's head, and his hands were left free for the manipulation of the taps. As mercury was introduced slowly into the apparatus the contact was well defined by a roaring due to the outstanding vibrations of the mercury surface, followed immediately by the continuous note of the trembling coil.

A general view of the apparatus is shown in fig. 3. The readings were made in the following manner:—By means of the hand Fleuss pump the vessel V was partially exhausted. It was then placed in communication with the index tube D, and the level of the mercury in that tube raised to about 3 cm. above the level at which contact had occurred during the previous experiment. The mercury in the contact tube was then below the pointer, and the telephone was silent. The two-way tap S was then turned, and air gradually admitted to the index tube D through the tube W, which was fine-drawn to prevent the thread of mercury from moving too rapidly. The mercury fell slowly in the index tube until the level of the mercury in the contact tube had been raised sufficiently for the telephone circuit to be completed. The first observation was a preliminary one, and the experiment was repeated. On this second occasion the level of the mercury in the index tube was raised only about 5 mm. above the point of contact given by the rough experiment, and the tap S opened to a small extent only, so that the level should fall exceedingly slowly. When the telephone indicated that contact was made, the reading of the mercury in the index tube was recorded. The process could be reversed by first lowering the level of the mercury in the index tube and then connecting up with the exhausted reservoir.

(We may note here that it would prove an advantage to include a second glass tap at the base of the index tube, corresponding in position to the stopcock L in the rough-adjustment tube, and to prolong the barometer and contact tubes to reach nearly to the bottom of the reservoir R, as in the Kew pattern marine barometer; the instrument would then be more portable.)

Details of the Design.

Having considered the general working of the apparatus, we proceed to discuss in detail the factors which govern the design of the instrument.

1. *The Contact Error.*—Before finally deciding to employ an electrical method for determining the moment of contact, experiments were carried out in the Physics Laboratory of University College, Reading, with the assistance of Dr. Sadler, to estimate the relative sensitivities of electrical contact and microscopic observations. A bell was in circuit with the contact, and the level of the mercury was raised gradually until it rang; this was done several times, and the level of the mercury at contact observed microscopically, but no difference between successive readings of the mercury level was discernible. As the use of a microscope attached to a swinging instrument presented difficulties, the contact method was chosen; it was subsequently greatly improved by substituting a telephone for the bell indicator.

The micrometer eyepiece of the microscope read to ·02 mm., and a change of level as small as ·002 mm. should have been easily detected, but the consistency may have been even greater. A platinum point was used on the outward and a steel point on the homeward voyage; the latter is preferable.

The experience gained at sea, where, for obvious reasons, the estimate of the instant of contact is more difficult than on land, and is liable to errors from pumping, vibration, etc., confirms the writer in the opinion that the method chosen is simple and satisfactory. When a steel point is used it would be well to fill the bulb with nitrogen instead of air.

2. *The Temperature Correction.*—The quantity of mercury necessary to give temperature compensation may be determined as follows:—If P and T are the pressure and temperature of the air in the reservoir,

$$\delta \mathrm{P}/\mathrm{P} = \delta h/h = \delta \mathrm{T}/\mathrm{T}.$$

A change δh in the height of the barometer column would involve a transfer of volume $a\delta h$ from the index to the barometer tube. If there is no change of level in the index tube, the increase in the volume of mercury due to the rise of temperature must be equal to $a\delta h$. Hence if v be the total volume of mercury, and α, the coefficient of dilatation, $\alpha v\delta \mathrm{T} = a\delta h = ah\mathrm{T}^{-1}\delta \mathrm{T}$, from which it follows that $\alpha v = ah/\mathrm{T}$.

For the test instrument the volume of the mercury should equal 79·6 c.c. for use at 4° C., and the instrument as originally constructed had a volume equal to 80 c.c. very nearly. Unfortunately its reconstruction in Australia involved the replacement of the bulb by another which had a capacity which was considerably greater than was necessary, and there was over-compensation.

3. *The Viscosity of the Mercury.*—The introduction of capillary tubing, while necessary to prevent "pumping," is liable to make the rates of flow of the mercury from the index tube into the barometer and contact chamber different in the two cases; contact will then be recorded too early or too late, and without the accurate balancing of the two columns.

Any error due to this cause may be reduced to a minimum by making the rate of flow of mercury constant in all experiments, and very small; during the trial it was the practice to find the approximate reading by a rapid preliminary experiment, and subsequently to approach this value slowly; but the manipulation could be improved by attaching a manometer to the vessel V, fig. 3, and always exhausting it to the same extent, and also by always opening the stopcocks by fixed amounts.

We may test the apparatus for the error introduced by the viscosity of the mercury by making successive readings (1) when the mercury is being extracted from, and (2) when it is being introduced into the instrument; the difference should give a measure of twice the extreme error introduced by the viscosity of the mercury.

Four pairs of observations enable us to test the behaviour of the above apparatus in this respect at sea. Two were made in Colombo Harbour, and two in the open sea before reaching that port.

Mercury in index tube.	Open sea.		Colombo Harbour.	
	Sept. 21.	Sept. 21.	Sept. 22.	Sept. 23.
	mm.	mm.	mm.	mm.
(1) Rising	127·7	111·8	118·1	171·9
(2) Falling	126·9	113·0	118·5	171·9
Divergence from mean	0·4	0·6	0·2	0

The average divergence from the mean for the harbour observations is 0·1 mm., which is equivalent to 0·006 cm./sec.^{2} in gravitational units. At sea the divergence is 0·029 cm./sec.^{2}; inexperience has something to do with the earlier readings being more divergent.

We may safely say that the errors introduced by viscosity are less than these limits, and probably much less, since all the above readings were subject to errors due to temperature changes, and the vibration of the mercury surface (the engines were throbbing during the harbour readings).

Greater consistency is to be expected when observations are made in the same way or when the mean of the readings with rising and falling mercury are taken. I can only find in my notes one instance of a pair of observations made in the same manner between which the temperature had not changed appreciably. They were made at sea, and there was only a difference between them of 0·3 mm., corresponding to a divergence from the mean equivalent to 0·009 cm./sec.^{2}, and here again pumping, contact, vibration, temperature, and viscosity errors are summed up.

Gravity determinations at sea all depend upon the measurement of the height of a constricted mercury column, and the lag is a source of error whose correction is a matter of uncertainty, especially in an open barometer, since it depends upon the vagaries of the atmospheric pressure at the time of the experiment. Here, however, we have a means either of reducing the lag to zero or at least of making it always the same by introducing the mercury at a constant rate. In this respect the instrument possesses an advantage over other types of marine barometers used for measuring gravity at sea.

Preliminary Test of the Apparatus at Sea.

In June, 1914, Messrs. Cossor constructed for the writer two pieces of apparatus of the above design, differing in certain details. They were carried to Australia in the Blue Funnel liner 'Ascanius,' in which, through the generosity of Messrs. Alfred Holt, a laboratory had been installed for the purpose of measuring the variations of gravity over the ocean. Two other methods of attack were attempted, and as they were more matured than the present method, the principal attention was devoted to them. The outward voyage was chiefly occupied in the acquisition of experience of fresh difficulties to be overcome; for instance, in the case of the apparatus now being described, it was found that a bell indicator would not work satisfactorily on a rolling ship; that the original gimbals mounting was unsuitable, that packing with cotton wool was not sufficient protection from the variations in the temperature of the refrigerator, and that a tank must be employed; that it was a mistake to put oil above the mercury to keep the contact clean, because the motion of the ship caused some emulsification, and some oil got into the capillary and broke the electrical circuit (as originally constructed the second terminal was sealed into the bulb of mercury); also that glass taps worked loose on board ship, even when provided with rubber bands, an accident which led to the inrush of air and the fracture of one of the instruments. Unfortunately the 'Ascanius' was requisitioned as a troopship on arrival in Australia, and it was not found possible to make full use of the experience gained. Nevertheless, the remaining apparatus was opened up, the oil removed, the contact chamber reconstructed—a requirement which unfortunately led to the substitution of a new for the original mercury reservoir—the tanks and their fittings made, the suspension altered, telephone indicator introduced, stirring arranged for as described in the foregoing account, the instrument refilled with mercury, and put on board the P. and O. R.M.S. 'Morea,' in whose refrigerating chamber accommodation had been found for the experimenter through the courtesy of the Superintendent in Sydney. For the first ten days of the voyage the temperature adjustment of the refrigerator and the other pieces of apparatus required the writer's undivided attention, but when within three days' voyage from Colombo all was in order, and a test was begun.

For four days observations were carried out, and it was discovered that the pumping had apparently been satisfactorily overcome, and that the telephone method was reliable and delicate, but unfortunately the temperature variations of the refrigerator laboratory on R.M.S. 'Morea' were beyond the control of the observer, and were unexpectedly large; moreover, in Colombo Harbour, from readings at different temperatures, the temperature coefficient of the instrument between 8° and 9° C. was found to be such that a rise of 1° C. necessitated the extraction of a thread of mercury 5·93 cm. in length, indicating considerable over-compensation. Satisfactory results could not be obtained with so large a correction factor, because a sufficiently delicate thermometer was not available, and the variation of g has not been determined.

As a single instance of the use of the instrument, even with so large a temperature correction (it may involve an inaccuracy of 0·1 cm./sec.^{2} in gravitational units), the following set of observations is appended (Table II). The readings have all been reduced to a temperature of 8·65° C., the mean of the temperatures of the two Colombo readings.

The value of Δg was calculated from the knowledge that the area of the barometer tube was 84·5 times that of the capillary index tube, and that the instrument was designed for a mercury column 20 cm. in height, where gravity has the value 981. Thus 1 mm. in the index tube is equivalent to a change in the value of gravity of 0·058 cm./sec.^{2} In the later observations a break in the thread of mercury in the capillary of the barometer tube was a great source of trouble, and observations were finally abandoned on account of it. It was occasioned by a minute sealed-up crack which the ship's motion reopened to an infinitesimal extent.

No claim is made that these results represent the behaviour of gravity over the range of latitude investigated. The Table is intended to illustrate the nature of the reductions of the readings of the instrument under examination. Nevertheless, the observations from 9.43 A.M. to 10.5 P.M. on September 22,

Table II.

Dates.	Time.	Temperature of tank.	Movable column.	Reduced to 8·65°.	Latitude.	Depth.	Δg.
1914.		° C.	cm.			fathoms.	cm./sec.^{2}
Sept. 21...	12.45 p.m.	7·70	8·31	13·95	1° 0′ N.	2750	−0·029
	5.0 „	8·00	11·30	14·15	2 8	2667	+0·087
	6.0 „	7·60	7·98	14·20	2 20	2608	+0·116
	6.25 „	7·50	6·50	13·32	2 25	2608	[−0·395]
	6.45 „	7·25	5·37	13·67	2 30	2617	−0·192
Sept. 22...	9.43 a.m.	8·70	13·64	13·93	5 46	Diminishing	−0·041
	9.48 „	Falling	13·80	14·09	5 48	760	+0·052
	9.56 „	8·65	14·13	14·13	5 50	234	+0·075
	10.5 „	8·6	13·85	14·14	5 52	36	+0·081
	The thread of mercury in the capillary of the barometer tube was observed to be broken.						
	10.28 a.m.	8·5	13·06	13·95	5 55	35	−0·029
	11.17 „	8·0	10·14	13·99	6 01	23	−0·006
	2.25 p.m.	7·1	3·90	13·10	6 48	27	−0·522
	2.45 „	7·25	5·46	13·76	6 53	21	−0·139
	5.51 „	8·20	11·85	14. 52	6 54	32 ft.	+0·302*
Sept. 23...	5.38 „	9·10	17·19	14·52	6 54	32 ft.	+0·302*
	8.52 „	7·60	7·78	14·01	7 2	532	+0·006
	9.21 „	7·40	6·88	14·29	7 4	782	+0·168
	10.0 „	7·50	7·02	13·83	7 7	1310	−0·099
	10.23 „	7·40	6·77	14·18	7 10	1425	+0·104
Sept. 25...	10.40 a.m.	8·60	13·42	13·72	14 40	22	[−0·162]
	10.55 „	8·45	13·10	14·28	14 44	27	+0·162
	1.2 p.m.	8·00	10·44	14·30	15 15	22	+0·174
	1.15 „	8·05	11·10	14·66	16 4	20	+0·382
	5.38 „	8·97	16·22	14·32	16 13	23	+0·186

Δg is the deviation from the theoretical value for g at the Equator.

* Colombo.

which were made when the ship was rapidly running into shallow water, probably represent a real phenomenon, because, though the temperature was falling, it was necessary to extract mercury. This indicates the increase of gravity as land was being approached. The high value at Colombo is also probable, and the general rise in the value of g as the Equator was left (fig. 4) is in accord with our knowledge of the variation of g with latitude, and between Lat. 0° and 16° N. is of the same order of magnitude as is demanded by Helmert's formula, namely, 0·39 cm., but too much stress must not be laid upon this on account of the temperature difficulty already mentioned. The very low value obtained off Adam's Peak is almost certainly accidental. In the diagram (fig. 4) shallow and deep water observations are distinguished from one another; there are no systematic differences in gravity attributable to this cause if the latitude variation be taken into account. These results obtained in the preliminary trials of the instrument justify its further use for future examinations of the variations of gravity over the oceans.

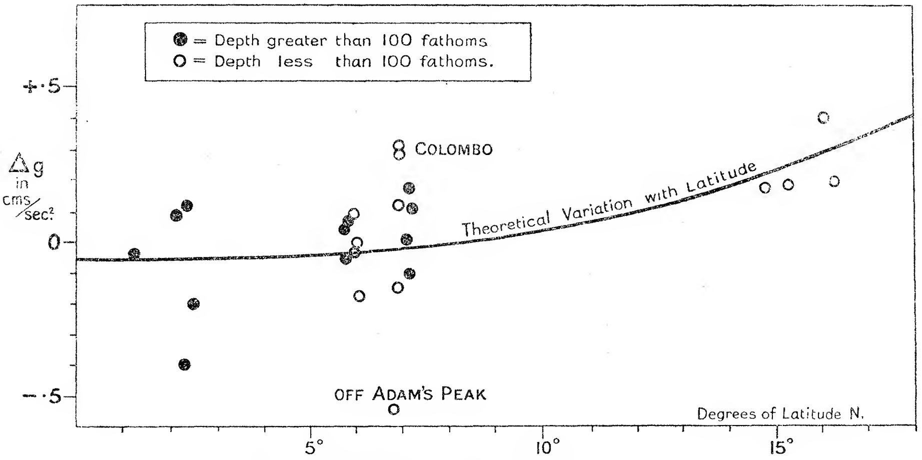

Fig. 4.—Preliminary test of the instrument.

Temperature Regulation on Board Ship.

The experiments on the outward voyage in s.s. 'Ascanius' demonstrate the practicability of maintaining an experimental chamber, 8 feet by 4 feet, within a limit of temperature of 1° C. for days on end provided that the observer does not stay in it for longer than, say, 15 min. at a time.

The experimental chamber had been generously installed by Messrs. Alfred Holt, of the Blue Funnel Line of steamships, within the main refrigerator of the vessel. It was furnished with a separate system of brine pipes served by the auxiliary pumping engine; the flow of brine was regulated by the refrigerating engineers at least once every hour of the day and night after the temperature of the chamber had been read from outside by means of a thermometer which pierced the walls. Towards the end of the voyage the engineers had become proficient in compensating by an extra flow of brine for the introduction of the observer's body. Under such fortunate conditions as these it should be possible to maintain the temperature of the tank sufficiently constant for the estimation of g at sea, even if the temperature coefficient has not been reduced to zero.

Part of the expenses of this research has been borne by the Government Grant Committee.

The writer's indebtedness to Mr. Horace Darwin and to Messrs. Alfred Holt, of the Blue Funnel Line, have been referred to in the text. Dr. Charles A. Sadler, of the Physical Laboratory of University College, Reading, rendered valuable service in superintending and assisting in the construction of the apparatus at Messrs. Cossor's works, and in helping to install it in the refrigerator of s.s. 'Ascanius' in Liverpool. These experiments could not have been undertaken had it not been for the help kindly given by Captain Chrimes, Chief Engineer Douglas, and Refrigerating Engineer Latham, of s.s. 'Ascanius,' and by the Superintendent of the P. and O. Company in Sydney, the Chief Officers, the Purser, and Mr. Charlwood, of R.M.S. 'Morea.' The writer is also indebted to Mr. C. C. Pearce for assistance with the diagrams.

It was through the hospitality of Prof. Kerr Grant, of the Physical Laboratory of the Adelaide University, and through the skill of Mr. Rogers, of the workshop there, that it was possible so to modify the apparatus that a test was possible on the homeward voyage.

On the Determination of Gravity at Sea (Note on Dr. Duffield's Paper).

By Arthur Schuster, Sec. R.S.

(Received June 28, 1916.)

1. In a paper recently communicated to this Society, Dr. Duffield has described a promising method of determining gravity at sea, but a somewhat fuller discussion on the theoretical side is required before an opinion of its capabilities and limitations can be formed. For this purpose it is more especially necessary to investigate the effects of the forced vibrations which may be imposed on the mercury in the apparatus by the vertical acceleration of the ship.

2. The problem of determining the period of free oscillation in a barometer tube is very simple, but in the present case three tubes are connected with each other, and at least two of the tubes are not of uniform section, but contain a portion with a narrow bore introduced to damp the oscillations.

Starting with a single tube (fig. 1), let a glass vessel closed at the top and dipping into a trough of mercury at its lower end consist of a series of tubes having different diameters. The upper part of the vessel is supposed to contain air at a reduced pressure, and the mercury is disturbed from its equilibrium position at H. In each section of the tube having a radius r_0, the velocity of the flow at a point which is at a distance r from the central axis is $v = v_0 (r_0^2 - r^2)/r_0^2$, where v_0 is the velocity along the axis. From this it follows that the mean velocity and the mean square of the velocity